Physical Chemistry Cumulative Exam: Photons and Electrons! Feb. 4, 2015
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Don't Panic

The Photon! (25%)

1. A photon has been experimentally determined to have a total angular momentum quantum number of ℓ=1. What is the actual value for the angular momentum? (i.e. how do you "use" the angular quantum number to determine the actual angular momentum? Hint: you better have proper units!)

2. Give an experimentally observable consequence of the fact that light has ℓ=1.

3. Given that light has ℓ=1, what are the possible m_ℓ values?

4. There are only two known forms of light: (ℓ,m_ℓ) = 1,-1 which is right circularly polarized light, and (ℓ,m_ℓ) = 1,+1which corresponds to left circularly polarized light.
Why does (ℓ,m_ℓ) = 1,0 not exist? Hint: The projection of angular momentum is in the same direction as the propagation of the photon for m_ℓ = 0.

5. We tell undergraduates that light is plane-polarized. Yet from #4 I see that light can only be right or left circularly polarized. How can you account for plane-polarized light then? Are we just lying through our teeth?

6. The energy of light is hc/λ (h is Plank's constant, c is the speed of light, and λ is the wavelength). Use Einstein's theory of relativity: $E^2 = p^2 \cdot c^2 + m^2 \cdot c^4$ (m is the rest mass and p is the momentum) to show what the momentum of light is.

The Electron! (25%)

7. This doesn't get more simple- let's say that the universe *only* consists of an electron at rest (i.e. kinetic energy (K.E.) = 0 J and momentum (p) = 0 kg·m/s), and a 400 nm photon.
(btw: equations and constants are provided on the last page!)

a) What is the energy and momentum of the 400 nm photon?
b) What is the electron's kinetic energy after absorbing the photon?
c) What is the electron's momentum after absorbing the photon?

8. A free electron in space cannot absorb light. Why not? Hint: look at your answers on #7c, and calculate the energy of the electron using $p^2/2m$. Compare that to answer #7b.

9. Can you provide a simple description of electronic spin?

10. Is an electron a fermion or a boson?

11. Let's say that we can separate the electron's spin wavefunction from the spatial wavefunction as so: Ψ|α>, where Ψ represents the spatial location of the electron and |α> is the "spin-up" wavefunction (and |β> is spin down).

a) While I know all about performing operations on spatial wavefunctions, what is the mathematical behavior of spin wavefunctions? For example, I know Ψ for the s-state of hydrogen is $\sim e^{-r/a_0}$. Can you mathematically express a spin wavefunction in the same way?

b) Please evaluate the following two spin overlap integrals:

1) $\langle \alpha \| \alpha \rangle$ **2**) $\langle \alpha \| \beta \rangle$

c) Calculate the expectation value of the spin operator $\hat{\mathbf{S}}$:

1) $\langle \alpha | \cdot \hat{S} \cdot | \alpha \rangle$ **2**) $\langle \alpha | \cdot \hat{S} \cdot | \beta \rangle$ (obviously show your work!)

Electrons! (25%)

12. Now let's say an atom has two electrons. Below are 6 possible wavefunction descriptions, including one singlet and three triplets. Two are fundamentally incorrect. Please identify each one below as belonging to either the singlet or one of the triplet states (as well as why), and which ones are not valid (state why too!).

a) Ψ(1)|α(1)> · Ψ(2)|β(2)>

b) [Ψ(1)Ψ(2) – Ψ(2)Ψ(1)] · (|α(1)>|α(2)>)

c) Ψ(1)Ψ(2) · (|α(1)>α(2)> – |α(2)>α(1)>)

d) Ψ(1)Ψ(2) · (|α(1)>β(2)> – |α(2)>β(1)>)

e) Ψ(1)Ψ(2) · (|α(1)>β(2)> + |α(2)>β(1)>)

d) [Ψ(1)Ψ(2) – Ψ(2)Ψ(1)] · (|β(1)>|β(2)>)

13. In the above examples, there are instances when the wavefunctions are multiplied and others where they are added. What does it mean if the total wavefunction for two particles (electrons or not…) are added [Ψ(1) + Ψ(2)] or multiplied [Ψ(1)·Ψ(2)], or does it mean anything at all?

14. If electrons are separately described by their orbital angular momentum (s, p, d etc.) and as being spin up (α) or spin down (β) (as in Ψ(1)|β(1)>), what does this say about the Hamiltonian in terms of the angular momentum and spin? What can mess up this description such that the wavefunction ***could not*** be described by angular momentum and spin separately?

15. Fermionic particles must have total wavefunctions that are antisymmetric with respect to interchange of the particles' coordinates; how does this lead to exchange energy? Can you name a physical phenomena described by (or is the result of) exchange?

Physical Chemistry History! Answer just two! (10%)

16. Describe what either Henri Moissan, Joseph Priestley, William Ramsay, or Glen Seaborg discovered (just pick one).

17. Lasers are very monochromatic and can be tuned to different wavelenghts. Yet, I can make tunable monochromatic light with a light bulb and a very long-path monochromator. So, what can I do with a laser that I can't do with a light bulb and a monochromator?

18. Recent history: describe the work of any chemistry speaker (include their name) this past year.

Safety! Answer just three! (15%)

19. **a**. What is a Lab Safety Plan? **b**. Do you know where your lab's safety plan is? (yes or no only!)

20. What is an MSDS?

21. Which is more dangerous, a class I or a class IV laser and why?

22. Which is more dangerous, nitric acid or hydrochloric acid and why?

This is all you need for the test.

Joule (J) = $kg \cdot m^2/s^2$

Speed of light (c) = 2.99792×10^{8} m/s

Plank's constant (h) = 6.62606896×10^{-34} J·s

h/2π (ħ) = $1.054571628\times10^{-34}$ J·s

electron mass = 9.10938188×10^{-31} kg

proton mass = 1.67262158×10^{-27} kg

neutron mass = 1.67492729×10^{-27} kg

photon mass = 0.00000 kg

photon momentum = h/λ

photon energy = hν = h·c/λ

frequency of light (ν) = c/λ

General Equations:

Angular momentum: $\mathbf{L} = \pm\hbar\sqrt{l(l+1)}$ m_ℓ : -ℓ, -ℓ+1, …0,… +ℓ

Kinetic energy (K. E.) = $\frac{1}{2}\, m\cdot v^2$

Fermion Wavefunction: $\Psi_1(1)\Psi_2(2) = -\Psi_1(2)\Psi_2(1)$

Momentum (p) = m·v

Boson Wavefunction: $\Psi_1(1)\Psi_2(2) = \Psi_1(2)\Psi_2(1)$

Spin operator $\hat{\mathbf{S}} = \pm\hbar\sqrt{s(s+1)}$ $\hat{\mathbf{S}}|\alpha\rangle = 0.5\hbar\,|\alpha\rangle$ $\hat{\mathbf{S}}|\beta\rangle = -0.5\hbar\,|\beta\rangle$

Picture of a pig in boots: