CUME for May 2015

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1. Suppose you have an atom with a spin quantum number I=1/2 (i.e. it can populate one of two nuclear spin angular momentum states, α or β.)

   a. (**10pt**) What is the size of the Hilbert space?

   b. (**10pt**) How would you determine the energy of these states?

   c. (**10pt**) The following are bra-kets: $|\alpha\rangle, \langle\beta|$. These are either eigenvalues, eigenfunctions, expectation values, wavefunctions, Hamiltonians or density operators of the system. Describe which one or more apply.

   d. (**10pt**) A spin is entangled into the following state: $\psi = 2^{-1/2}|\alpha\rangle + 2^{-1/2}|\beta\rangle$ in the z-axis eigenbasis. If the wavefunction is collapsed in a magnetic field aligned along the z-axis, what are the expectation value(s) and eigenvalue(s) of the system if thousands of atoms are measured in sequence?

   e. (**10pt**) After projecting and separating the atoms based on spin state from the question (c) above, the atoms $|\alpha\rangle$ and $|\beta\rangle$ are *seperately* subjected to a magnetic field along the x-axis. How will this influence the expectation value(s) and eigenvalue(s)? The z-axis and x-axis spin angular momentum states are non-commutable and hold a Heisenberg uncertainty relationship.

   f. (**10pt**) Write down the matrix representation of the following: $|\alpha\rangle$, $\langle\beta|$, $\langle\alpha|\beta\rangle$ and $|\beta\rangle\langle\alpha|$.

   g. (**10pt**) How would you predict how the wavefunction $\psi = 2^{-1/2}|\alpha\rangle + 2^{-1/2}|\beta\rangle$ will collapse?

   h. (**10pt**) Several theories have been postulated on the precise time or occasion in which a wavefunction collapses. One such theory is Schrödinger's cat. Can you briefly (2-3 sentences) describe this experiment and explain who, what, when or how the wavefunction is collapsed?

   i. (**10pt**) Can you write an arbitrary wavefunction composed of the eigenstates of two spins with I=1/2?

   j. (BONUS: **10pt**) Cryptography makes use a randomized keys, ideally with high entropy. Indeterminism of an entangled system as well as the destructive nature of wavefunction collapse (quantum decoherence) make these ideal systems for cryptography. Can you explain at least one of these features?