

# Don't Panic

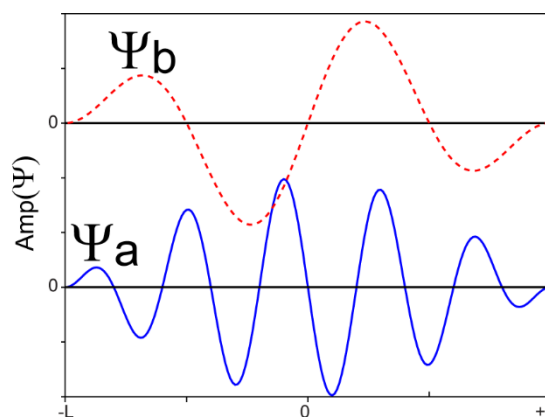
## Quantum Basics

45%

1. Why do things have quantized energy levels? Note: "Due to quantum mechanics" is not a good enough answer; for example, the Schrodinger equation correctly predicts that a particle in free space with no boundaries doesn't have quantized energy levels.

2. Without any additional information, can you solve the following integral for a normalized wavefunction?  $\int_{-\infty}^{\infty} \Psi(x) \partial x = ?$

3. Which wavefunction (**a** or **b**) corresponds to a faster moving particle and why?



4. a. There is an uncertainty relationship between position and momentum. Is that the only uncertainty relationship in Quantum Mechanics? If not, can you provide another example?

b. What is a Hermitian operator?

5. Which of the following functions are eigenvectors of the momentum operator  $\hat{p}_x = \frac{\hbar}{i} \frac{\partial}{\partial x}$ ?

Prove your answer.

a.  $k \cdot x$

b.  $\sin(k \cdot x)$

c.  $e^{-ikx}$

d.  $e^{ikx} + e^{-ikx}$

e.  $e^{-k \cdot x^2}$

## Quantum Hard-ics

45%

6. For the following Hamiltonian:  $\frac{-\hbar^2}{2m} \cdot \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right)$ , tell me what the energy of this state is:  
 $\psi(x,y) = \sin(k_x \cdot x) \cdot \cos(k_y \cdot y)$ .

7. The kinetic energy operator  $\frac{p^2}{2m}$ , which is equal to:  $\frac{-\hbar^2}{2m} \cdot \frac{\partial^2}{\partial x^2}$ , was designed to be consistent with the deBroglie relationship:  $p = \frac{h}{\lambda}$ .

Can you show that to be the case with  $\psi(x) = \sin(2\pi \cdot x/\lambda)$ ? Hint: operate on the wavefunction and solve for  $p$ , and note  $\hbar = \frac{h}{2\pi}$ .

8. Here are the wavefunctions for three particles:

$$\psi_A(x) = e^{-i \cdot x} \quad \psi_B(x) = \sin(x) \quad \psi_C(x) = e^{i \cdot x}$$

The flux operator acting on a wavefunction is evaluated as:  $\frac{\hbar}{2m \cdot i} \left( \psi^* \frac{\partial \psi}{\partial x} - \psi \frac{\partial \psi^*}{\partial x} \right)$

Now tell me whether the particles are moving left, right, or not at all.

9. Why would I need to use perturbation theory? Try to describe how it works in general terms, and show me how to solve a wavefunction using the set of unperturbed eigenstates:  $\Psi_n(x)$  where  $n$  is an index that goes from:  $n=1 \rightarrow \infty$ .

10. Which of the following is a valid wavefunction for a helium atom ground state and why? Note: the 1,2 denote the electrons,  $\alpha$  is the spin up wavefunction, and  $\beta$  is the spin down wavefunction.

a)  $\psi_{1s}(1) \psi_{1s}(2) \cdot (\alpha(1)\beta(2) - \beta(1)\alpha(2))$       b)  $\psi_{1s}(1)\alpha(1) + \psi_{1s}(2)\beta(2)$

c)  $(\psi_{1s}(1) \psi_{1s}(2) - \psi_{1s}(2) \psi_{1s}(1)) \cdot (\alpha(1)\alpha(2))$

## Safety

10%

11. A safety inspector is knocking on your lab door. What should you do?

12. What number do you call if there is an immediate emergency?

13. Can you pour some methanol down the sink?

14. Does Illinois OSHA have any jurisdiction over you if you're a theorist and work in an office?

15. What is Illinois OSHA?

## Extra Information

Constants:  $h = 6.626 \times 10^{-34} \text{ J} \cdot \text{s}$        $\hbar = \frac{h}{2\pi} = 1.055 \times 10^{-34} \text{ J} \cdot \text{s}$        $k_B = 1.38 \times 10^{-23} \text{ J/K}$

### General Equations:

$$\frac{\partial}{\partial x} e^{a \cdot x} = a \cdot e^{a \cdot x} \quad \text{deBroglie wavelength: } \lambda = \frac{h}{p} \quad \text{Eigenvalue equation: } \hat{\Omega}\Phi = \omega\Phi$$

Expectation value :  $\langle \hat{\Omega} \rangle = \int_{\text{all limits}} \Phi^* \cdot \hat{\Omega} \Phi = \int_{\text{all limits}} \Phi^* \cdot \omega \Phi = \omega \int_{\text{all limits}} \Phi^* \Phi = \omega$

Operators (1-D): momentum  $\hat{p}_x = \frac{\hbar}{i} \frac{\partial}{\partial x}$       position  $\hat{x} = x$

kinetic energy  $\frac{-\hbar^2}{2m} \frac{\partial^2}{\partial x^2}$       Hamiltonian  $\frac{-\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V$

Hydrogen radial Hamiltonian:  $\frac{-\hbar^2}{2m_e} \frac{1}{r} \frac{\partial^2}{\partial r^2} r + \frac{\hbar^2 \cdot l(l+1)}{2m_e \cdot r^2} - \frac{e^2}{4\pi\epsilon_0 r} + c \cdot \hat{l} \cdot \hat{s}$  (c is a constant)

Hydrogen radial wavefunctions:  $N \cdot r^l \cdot (\text{connector}) \cdot e^{-c \cdot r}$  (N is the normalizer, c is a constant)

Hamiltonian for  $2e^-$ :  $\frac{-\hbar^2}{2m_e} \nabla_1^2 - \frac{e}{4\pi\epsilon_0 r_1} - \frac{\hbar^2}{2m_e} \nabla_2^2 - \frac{e}{4\pi\epsilon_0 r_2} + \frac{e}{4\pi\epsilon_0 |r_1 - r_2|}$

### Spectroscopy:

$$\Delta\mu = \int \Psi_f^* \cdot \mu \cdot \Psi_i \partial\tau \quad \partial\tau = r^2 \sin(\theta) \partial r \partial\theta \partial\phi$$

X-ray ← UV ← Visible ← Infrared ← Microwave ← Radio

**Rotation:**      Linear Rotor:  $E = B \cdot J(J+1)$       Symmetric Rotor:  $E = B \cdot J(J+1) + (A-B) \cdot K^2$

**Vibration:**       $V(x) = D(1 - e^{-c(x-x_0)})^2$        $E(v) = (v + 1/2)\hbar\omega - c(v + 1/2)^2 \hbar\omega$